The Framework for Teaching Clusters

Six Clusters Supporting High Level Learning

Math Version

The Danielson Group

The Framework for Teaching: Six Clusters Supporting High Level Learning

The *Framework for Teaching Clusters* provide a description of the skills demonstrated by accomplished teachers in promoting high levels of student performance—skills based on foundational knowledge and dispositions and grounded in a deep understanding of the nature of human learning. The Clusters are an outgrowth of *The Framework for Teaching* (the FfT), which has been validated through empirical studies as predictive of student learning as measured by state assessments. But while the FfT has enjoyed wide acceptance among members of the professional community of educators, its level of detail can make it cumbersome for everyday use. The FfT Clusters are an attempt to distill the "big ideas" of the FfT's four domains and 22 components into an efficient tool (composed of six large concepts) that can serve as the foundation for many purposes, most importantly— professional growth by teachers, not only through their own reflection on practice, but also through their conversations with colleagues, mentors and coaches, and supervisors.

The Clusters—like the full Framework—are themselves generic in nature; that is, they apply to all teaching situations, in all disciplines and at different ages and levels. Furthermore, they reflect teaching to high standards of student learning, as reflected in the Common Core State Standards and other high-level standards. Some of these principles of teaching for CCSS learning are, indeed, generic. For example, teaching for deep conceptual understanding, the use of precise academic language, and the skills of argumentation are evident in all disciplines. Similarly, student skill in questioning the reasoning of classmates, and their perseverance with challenging content occur in all settings.

On the other hand, teaching occurs in real settings, with real students, and about specific content. Therefore, while there is a generic skill of argumentation, for example, it plays out differently in mathematics than in literacy. Hence, The Clusters document is offered in several versions: a generic version, and separate versions for literacy and mathematics. Literacy skills are evident not only in English classrooms for literary analysis, but also in other disciplines, such as social studies and science, for reading for meaning. These versions translate the generic language of the narratives and critical attributes, where appropriate, into content-specific language to guide both teachers and leaders.

Furthermore, while the FfT Clusters – like the full Framework for Teaching – reflect teaching practices that are common across all settings, actual teaching occurs with students in all their diversity – cultural, linguistic, and developmental. Hence, accomplished teachers must be familiar with their students' individual characteristics and needs, and create their plans and provide instruction accordingly. Therefore, when the language of the Framework refers to attending to individual students, it is to this full range of learners that it applies. These are the "Common Themes" of the Framework for Teaching, which permeate all the components, and elements, and ensure an inclusive environment for learning.

The mathematics version of the FfT Clusters, reflecting those instructional practices that are manifested in mathematics classrooms, comprises the remainder of this document. The practices reflect certain perspectives on the learning of mathematics that, while they share some elements with other disciplines, are particular to mathematics.

Proficiency in mathematics involves both content and process, conceptual understanding and fluency with mathematical procedures, problem solving and mathematical reasoning. These aspects of mathematics learning are not independent of one another, but require the artful combination of challenge (for students to tackle unfamiliar problems), support (to encourage perseverance in tackling difficult challenges), and relentless focus on thinking (for students to develop skill, and confidence in their own capacity for understanding.) While every lesson has specific instructional purposes, students' knowledge and fluency in both mathematical content and processes develops through a series of encounters with increasingly sophisticated mathematical concepts, with investigations to deepen

understanding combined with mindful practice. The very phrase "habits of mind" suggests that it takes time to develop such mathematical habits and increased sophistication in content and, therefore, the objectives of any single lesson are embedded in larger goals that are developed gradually.

In addition, there is another aspect of mathematics teaching that warrants attention, having to do with the relationship between conceptual understanding and procedural knowledge. When number "facts" (for example, 6 + 8 = 14) are first introduced to students, they are grounded in the concepts of "6" and "8" and "14," and the concept of addition. That is, they're not yet understood as *facts.* Later, when the concepts underlying them are understood, they become facts to be remembered, and it's inefficient to continually think about them, or figure them out. Likewise, once a procedure is settled business, it becomes a tool in a toolkit; the ongoing instructional challenge is to help students know when best to use it, but not primarily to return to conceptual questions of the sort that were at play when the procedure was still new material. Thus, mathematics teaching presents a challenge for teachers, in that some teachers are tempted to teach students "tricks" to arrive at a correct answer, short-circuiting understanding. These strategies can be effective in the short run (they do, after all, enable students to get the right answer), but as a long-term approach their use is both limited and can actually impede future learning. Tricks and mnemonic devices generally only work in specific situations; to the extent that students don't understand the underlying mathematical concepts, they struggle when confronted with future learning.

For those familiar with the *Framework for Teaching*, the following table summarizes the relationship between The Clusters and the full FfT, together with the ways in which teachers might demonstrate their skill for each one. Sources of Evidence are provided for guidance, but the lists are not definitive. Not every artifact may be available. Quality evidence provides raw data for meaningful, professional conversations.

The Link between the Six Large Component Clusters and the Full Framework for Teaching

Cluster	FfT Components/Elements	Sources of Evidence
1. Clarity of Instructional Purpose and Accuracy of Content • *To what extent does the teacher demonstrate depth of important[1] mathematical conceptual content knowledge and conduct the class with a clear and ambitious purpose, reflective of both the content and process standards and appropriate to the students' levels of knowledge and skill?* • *To what degree are the elements of a lesson (the sequence of topics, instructional strategies, mathematical representations, and materials and resources) well designed and executed, and aligned with the purpose of the lesson? To what extent are they designed to engage students in high-level mathematical learning?* • *To what extent did the teacher make adaptations to the lesson?* • *To what extent did the teacher use formative assessment to check for student understanding?*	• 1a, 1b, 1c, 1d, 1f: Knowledge of content, clarity, and appropriateness for students of instructional outcomes; resources for classroom use, assessments aligned to instructional outcomes • 1e: Planned tasks to promote fluency, reasoning and problem solving, aligned to the instructional purpose • 3a: Expectations for learning, accuracy of content, clarity of explanations, use of academic language • 3b, 3c: Questions and discourse to encourage student thinking and problem solving, activities, tasks, and assignments aligned to mathematical purposes for both conceptual understanding and processes • 3d: use of formative assessments aligned to instructional goals	• Planning documents: learning outcomes, instructional activities • Observation: o Statements to students about purpose, conversation with students o Accuracy of mathematical concepts and practices o Alignment of questions, activities, and assignments to purpose • Reflection: success in facilitating the lesson objectives?
2. Safe, Respectful, Supportive, and Challenging Learning Environment • *To what extent do the interactions between teacher and students, and among students, demonstrate genuine caring and a safe, respectful, supportive, and also challenging learning environment? Does the teacher convey high expectations for student learning and encourage hard work and perseverance? Is the environment safe for risk taking? Do students take pride in their work and demonstrate a commitment to mastering challenging mathematical concepts?*	• 2a: All elements • 2b: Expectations for learning and achievement, student perseverance in challenging work, and pride in that work	• Observation: o Interactions of students and teacher o Student perseverance and pride o Students discussing and analyzing incomplete and wrong answers respectfully and productively • Student surveys

[1] In a college- and career-ready framework, the essential content of elementary grades math is number and operations, and knowing how the ideas of number and operations connect to one another and develop across the grades; in middle school, building ratio and proportion, pre-algebra, and algebra on what students know from arithmetic; and in high school, algebra, functions, and modeling applications. (Because these content areas form the core trajectory from early grades to readiness for college and careers, every effort should be made, when using the Framework, to observe lessons that target these content areas (at the appropriate level, given the stage of development of the students in the class).

Cluster	FfT Components/Elements	Sources of Evidence
3. Classroom Management • *Is the classroom well run and organized? Are classroom routines and procedures clear and carried out efficiently by both teacher and students with little loss of instructional time? To what extent do students themselves take an active role in their smooth operation? Are directions for activities clearly explained so that there is no confusion? Do students not only understand and comply with standards of conduct but also play an active part in setting the tone for maintaining those standards? How does the physical environment support the learning activities?*	• 2c: All elements • 2d: All elements • 2e: All elements	• Observation: o Routines o Student conduct o Physical environment
4. Student Intellectual Engagement • *To what extent are students intellectually engaged in a classroom of high intellectual energy? What is the nature of what students are doing? Are they being challenged to think and make mathematical connections through both the instructional activities and the questions explored? Do the teacher's explanations of concepts use meaningful representations and model mathematical language? To what extent are students engaged in mathematical discourse, and asked to explain their thinking, constructing logical arguments citing evidence, and questioning the thinking of others? To what extent do instructional tasks promote student agency in the learning of challenging mathematical content?*	• 1e: Design of instruction • 2b: Importance of the content • 3a: Explanations of content: their rigor and invitations for thinking • 3b: Quality of questions/discussions, student discourse • 3c: Intellectual challenge	• Planning documents • Observation: o The nature of the work students are doing o The quality of teacher presentation of content o The nature of student discourse and class discussion • Student worksheets or activities • Samples of student work
5. Successful Learning by All Students • *To what extent does the teacher ensure the learning by all students? Does the teacher monitor student understanding throughout the lesson through specifically designed questions or instructional techniques (such as monitoring tools and exit tickets)? To what extent do students monitor their own learning and provide respectful feedback to classmates? Does the teacher make modifications in presentations or learning activities where necessary, taking into account the degree of student learning? Has he or she sought out other resources (including parents) to support students' learning? In reflection, is the teacher aware of the success of the lesson in reaching students?*	• 1b: Knowledge of students • 1d: Resources for students • 1f: Design of summative and formative assessments aligned to outcomes • 3d: Monitoring of student learning, feedback to students, student self-assessment • 3e: Persistence, lesson adjustment • 4a: All elements • 4b: All elements • 4c: All elements	• Planning documents for formative and summative assessments • Observation: monitoring, feedback, adjustment • Reflection: comments on learning of individuals • Artifacts documenting both record keeping and communication with families

Cluster	FfT Components/Elements	Sources of Evidence
6. Professionalism • *To what extent does the teacher engage with the professional community (within the school and beyond) and demonstrate a commitment to ongoing mathematical professional learning? Does the teacher collaborate productively with colleagues and contribute to the life of the school? Does the teacher engage in mathematical professional learning, and take a leadership role in the school to promote the welfare of students?*	• 1d: Resources to extend professional knowledge • 4d: All elements • 4e: All elements • 4f: All elements	• Artifacts documenting contributions to a professional culture, to professional learning, and to other professional activities

Cluster 1: Clarity of Instructional Purpose and Accuracy of Mathematical Content, both concepts and processes

Teaching is a purposeful activity; it is goal directed and designed to achieve particular well-defined ends. Even when operating within the confines of an established curriculum (as virtually all teachers are), teachers must determine the purposes for a given class on a given day. In mathematics, those daily purposes are embedded in larger goals, which develop over time. That is, important mathematical understanding, for example, that of complex concepts and the skills of constructing chains of reasoning, do not lend itself to a single day's lesson, and is not "checked off" as complete. Understanding may develop slowly, based on a trajectory of learning, with the purpose for a given day anchoring a longer sequence of lessons that include the development of strategic thinking, and engaging in mathematical discourse, as well as developing fluency with the routine processes that inform more complex thinking. Daily purposes are important not only in themselves but because they form the foundation for more advanced mathematical proficiency.

Mathematical thinking and problem solving involve both content and processes. Students acquire these both independently and together, and well-designed lessons include elements from both. While every lesson has specific instructional purposes, students' knowledge and fluency in both mathematical content and processes develop through a series of encounters with those concepts, with investigations to deepen understanding combined with mindful practice. (The very phrase "habits of mind" suggests that it takes time to develop such mathematical habits and increased sophistication in content. Therefore, on the one hand, it is essential for teachers to be able to demonstrate clarity of daily instructional purpose. On the other hand, those purposes may not be the type that can be considered "finished."

Clarity of instructional purpose is essential to good teaching; classroom time is, after all, limited, and available time must be used wisely. Instructional purposes are statements, then, of what the teacher intends for students to learn; they should be clear and appropriately challenging for the students in the class. It is not sufficient for a teacher to state what the students will *do* during a lesson; he or she should also be clear about what they will *learn based on the trajectory of learning.* Admittedly, the learning outcomes are realized for students through the tasks and investigations in which they engage, but these activities and tasks must be designed such that they serve the teacher's instructional purpose.

Clarity of purpose implies alignment with the state's or district's curriculum outcomes (the Common Core or other high level standards), consisting of the factual, conceptual and procedural knowledge, skills, and understandings identified in the standards as well as the strategies and processes that relate to and underlie these skills and understandings. The content should be challenging and rigorous, and also appropriate for the students in the class; this suggests that learning outcomes may have to be individualized, to some degree, to enable all students to access the curriculum and to accommodate different students' backgrounds, prerequisite understanding, language proficiency, and special needs.

Such clarity of purpose requires deep knowledge of mathematics, of subject-specific pedagogy, and of one's students. Deep knowledge of content (as distinct from superficial familiarity) includes the teacher's understanding of the "big ideas" of mathematics, particularly those most essential for college and careers, and of how these are related to other important concepts, both within mathematics and in other disciplines. This deep knowledge of mathematics is revealed in many ways: the manner in which concepts are explored and explained, and in the teacher's response to student comments and questions. Indeed, a knowledgeable teacher will know whether a student's question is important to the discipline and therefore worth pursuing in depth, or whether it represents a sidebar and can be handled immediately. A knowledgeable teacher will also be able to recognize the important mathematics in students' thinking and make them explicit to the whole class.

Clarity of purpose also implies knowledge of essential prerequisite understanding (that is, which concepts depend on which others), flexibility of thinking, and recognition that there are many pathways to understanding. In planning a lesson, a teacher should be clear about the sequence of activities and tasks that will lead to student understanding. It is not sufficient that an activity is fun; it must also serve an important instructional goal.

A lesson's activities, as revealed both in the planning documents and in their execution in the classroom, must serve to achieve the lesson's purpose. In a well-designed lesson, these tasks and activities are sequenced and are designed to engage students in the intellectual work of learning. Furthermore, "clarity" extends to the activity itself. Students should not be in the dark about how to complete an activity, what steps they should take, and whether it's to be done on their own or with classmates, and how learning will be assessed. Instructive assessments will be grounded on the clarity of instructional purposes and the accuracy of content. The use of assessments is a focus of Cluster 5.

Well-run classrooms are purposeful and businesslike; they may be joyful, but students and teachers are clear not only about what they are doing, but also

about what desired learning is being pursued. There is a sense conveyed, through both words and actions, that what's going on in the lesson is important and that learning is exhilarating and empowering. Serendipity may permit the extension of the learning into other areas, but the fundamentals are clear and are grounded in the teacher's deep knowledge of mathematics and of the ways to engage students in exploring the many important concepts and relationships embedded in it.

Teachers also demonstrate their knowledge of content through their reflection on and analysis of the lesson. By identifying those portions of the lesson that were successful (while other portions were less so) and the reasons for these discrepancies, they demonstrate their understanding of the internal connections between different aspects of the content and how student learning can be assured.

Cluster 1: Clarity of Instructional Purpose and Accuracy of Mathematical Content, both concepts and processes			
Indicators: • Clarity of instructional outcomes, reflecting not only knowledge of content and of CCSS or other high-level standards and practices, but also suitability for the students in the class (1a, 1b, 1c) • Instructional outcomes reflecting both content and practices, concepts and skills, the relations between them, and strategic use of each (1c) • Lesson focus on a mix of conceptual understanding, application, and mathematical fluency (1c) • Planned resources and activities aligned to the instructional purpose (1d, 1e) • Expectations for learning, accuracy of content, development of practices, clarity of explanations, and use of academic language and mathematical terminology (3a) • Activities and assignments, questions and student discussion, all aligned to the instructional purpose (3b, 3c)		*Evidence*: • Planning documents, which state the mathematical goal, instructional purpose and planned activities • Observation: o Statements to students about purpose; conversations with students o Accuracy of mathematical concepts and fidelity to clear processes o Alignment of activities and assignments to the purpose o Questioning sequences that demonstrate a deep understanding of the content of the lesson • Reflection: success in achieving the lesson's objectives	
Unsatisfactory	**Basic**	**Proficient**	**Distinguished**
The instructional purpose and learning tasks are unclear; the information presented is inaccurate or inappropriate and unsuitable to the students, through some combination of the following: • *The teacher's plans indicate weak mathematical content knowledge.* • *The teacher does not try to ascertain varied skill and ability levels among students in the class to use in planning.* • *Learning outcomes, as stated by the teacher, are poorly aligned to the learning standards and either lack clarity or are stated as activities. They are unsuitable for a number of students in the class.* • *At no time during the lesson does the teacher convey to the students what they will be learning.* • *The teacher makes a serious mathematical content error or mistake in the use of mathematical language that will affect students' understanding of the lesson.* • *Students indicate through body language or verbal exchanges that they don't understand the mathematical content being presented.* • *Students appear confused about the learning task.*	The instructional purpose and learning tasks are somewhat clear; the information presented is primarily accurate and partially appropriate to the students, through some combination of the following: • *The teacher's plans reflect surface-level or purely procedural mathematical understanding without meaningful linkages to underlying concepts.* • *The teacher is aware that there are different skill and ability levels in the class but does not use this information in planning.* • *Learning outcomes, as stated by the teacher, are a combination of outcomes and activities or lack clarity; they are only partially aligned to the targeted learning standards, and are unsuitable for some students in the class.* • *The teacher refers in passing to what the students will be learning, or it is written on the board with no elaboration or explanation.* • *The teacher makes no serious mathematical content errors but may make minor ones (with no acknowledgement or correction), including imprecise use of academic mathematics language.* • *The teacher finds it necessary to clarify the learning task so that students can complete it.*	The instructional purpose and learning tasks are clear, the information presented is accurate and suitable to the students, through some combination of the following: • *The teacher's plans reflect important mathematical concepts and practices and their relationships to one another.* • *The teacher's plans demonstrate awareness of possible student misconceptions and how they can be addressed using formative assessment materials and practices.* • *The teacher has identified broad skill groups of students within the class and uses this information in planning.* • *Learning outcomes, as stated by the teacher, are written in the form of student learning and are aligned to the learning standards. They are suitable for the groups of students in the class.* • *The teacher states clearly, at some point during the lesson, what the students are learning.* • *The teacher makes no mathematical content errors and models the correct use of academic language and engagement in mathematical sense making* • *The teacher's explanation of the mathematics content is clear and invites student*	The purpose and learning tasks of the lesson are very clear, through some combination of the following, in addition to elements listed under "Proficient": • *The teacher's plans cite intra- and interdisciplinary relationships for mathematics outcomes.* • *The teacher is aware of the proficiency level of each student in the class and incorporates this understanding into plans.* • *When the teacher explains mathematical concepts, students generate predictions and make connections* • *The teacher's carefully-crafted questions enable students to extend the lesson's objective for deeper understanding.* • *The teacher is able to explain fully and specifically how planned learning tasks, materials, and problem sequences are well suited for the particular goals and topics of the lesson, unit, or longer-term work, and to the students in the class.* • *Planned activities provide students extended opportunities to develop and connect mathematical concepts, and to build productive mathematical habits of*

Cluster 1: Clarity of Instructional Purpose and Accuracy of Mathematical Content, both concepts and processes			
• *Planned learning tasks, materials, and question sequences are of low cognitive challenge, are unrelated to the lesson's stated purpose, or are not suitable for many students. Planned practice exercises to support learning are inappropriate to the learning goals.*	• *Planned learning tasks, materials, and question sequences are of moderate cognitive challenge or are only partially related to the lesson's stated purpose, or both. In particular, they provide little opportunity for mathematical sense making – for problem solving, producing and critiquing extended chains of reasoning, for using mathematics to model real-world situations.* • *Planned practice exercises to support learning are haphazardly designed or inappropriate in number (insufficient or else needlessly extensive).*	*participation and thinking.* • *Students engage with the learning task, indicating that they understand what they are to do; if modeling the process to be followed in the task is appropriate, the teacher does so.* • *Planned learning tasks, materials, and questions provide opportunities for mathematical sense making – for problem solving, producing and critiquing extended chains of reasoning, for using mathematics to model real-world situations.* • *Planned practice exercises to support learning are thoughtfully designed and appropriate in number.*	*mind (perseverance, sense making, problem solving, reasoning, reflection)* • *Planned learning tasks and materials permit advanced students to extend the lesson's purpose and provide students who need it most with more time, attention, and supports.*

Cluster 2: Safe, Respectful, Supportive, and Challenging Learning Environment

In order to do their best work and in order to make a commitment to the activity we call school, students must feel respected and honored as people. They must sense that their teachers believe in their capabilities; many adults can trace their success in school and in later years to a teacher who believed they could *be* somebody. For some students, this teacher may be the first, or the only, adult who has conveyed such confidence. It can be life altering.

Teachers convey their respect for students through a myriad of verbal and nonverbal cues, listening carefully to students' ideas, asking for clarification and elaboration, displaying sensitivity to students' feelings. A teacher's attitude may be outwardly friendly or stern, but beneath even a stern demeanor a teacher conveys an essential *caring*, a sense that each student, regardless of background or family circumstances, is important and has potential. Thus, students need not fear that they will be belittled by the teacher or demeaned in front of other students.

The atmosphere of support and respect is not confined to students as people but extends to them as learners of mathematics. Many adults are convinced that they "can't do math" or "were never good at reading poetry." While it's difficult to know the origin of such sentiments, teachers should never convey them. Thus, when teachers indicate that they sincerely honor all students in their journey for mathematical understanding, then students can engage in that quest assured of deep support by the teacher. It's a safe environment, in other words, for students to take intellectual risks, to try out ideas, to question the teacher's—or the book's, or another student's—account. Students need to know that their thinking is more important than the solution, and not fear ridicule, or unkind sarcasm, from the teacher or from other students. As the emotional environment clearly signals support and respect, the physical environment also conveys that learning is important and is rich and inviting.

While feeling safe with the teacher and other students will encourage their best work; students must also feel challenged, and they must be willing to rise to that challenge. This is partly a matter of the nature of the work itself; that work must be rigorous, engaging, and meaningful. But in addition, students must be willing to make a commitment to it. There must be, in other words, a prevailing norm of student commitment to high-level work; those who engage in such work must not be regarded by their classmates as "geeks," or "nerds," or some other term that, in student culture, denotes "un-cool." Furthermore, just as a classroom culture should honor intellectual work, that same culture should insist that students persevere with challenging mathematics, sticking with it until they "get" it and have achieved a higher level of understanding.

Student cultural attitudes toward work vary profoundly from one age group, and from one school, to another. Overwhelmingly, young children are keen to learn and to explore the world; if instructional tasks are interesting, then they will participate willingly and aim to excel. With older students, the situation is more complex; most of the efforts these students must make to succeed in school, after all, take place in private— for example, completing their homework assignments and studying for tests. But other actions occur in public, in front of their peers, such as participating in class discussions and engaging in group work. Thus, students who decide to make a commitment to high-level work in school are making a public declaration of that commitment. It's essential that they not become isolated or "punished" by their peers for that commitment.

In some settings, student norms already expect such commitment, for example, schools in communities whose families appreciate the importance of a rigorous education to ensure a successful future, or schools that have made a serious commitment to creating a culture for learning. But in other settings, particularly schools serving students of poorly educated families, the challenge for educators is far greater. Students' parents may themselves not have experienced the benefits that accrue from a solid education and from further study beyond secondary school. Parents may set expectations for their children's future based on their limited access to classrooms.

Educators have recently become aware of the powerful research regarding student mindsets, that is, how students view the role of intelligence in learning, (whether it's regarded as fixed or malleable), and the extent to which student success is a function of their views on the interaction between intelligence, on the one hand, and effort and hard work on the other. Thus, when students have become convinced that they're just "not good at math" they may be inclined to give up easily when a concept is difficult to understand. And if students have learned the "just not good at math" mindset from their families, such attitudes can be even harder to overcome. Researchers and teachers have found that to the extent that students acquire a growth (rather than a fixed) mindset, the more capable they are of both working hard and persevering through the inevitable difficulties all learners encounter in mastering complex material. Thus, teachers have an obligation to encourage such a growth mindset in their students, even

when it goes against the grain of what students think they know about their "natural ability" to learn mathematics.

Teachers whose classrooms constitute a safe and challenging environment for student learning have artfully combined challenge with support. They know their students well enough to know when a student has "blown off" an assignment, or when, in contrast, the student simply does not understand a concept well enough to complete high-quality work. When it comes to student commitment to learning, teachers don't take no for an answer, yet they are ready to provide necessary assistance when that's what's needed. This teaching is not formulaic; it is a high-level professional enterprise in which teachers know when to cajole, when to reteach, when to praise, and when to enlist the participation of other students—all in the service of high-level learning within an environment of challenge and support. Within this environment, students persevere in their quest for deep understanding and mastery.

A specific tool used by many teachers for ensuring high-quality work, and for enlisting students in the effort to engage everyone in the work at hand, is to teach students the skills of group work. After all, much important work in problem solving is best done in small groups – understanding what a problem is asking, brainstorming possible approaches, trying different solutions – and such group work, in order to be productive, requires important skills. These skills include, for example, listening to and respectfully disagreeing with others, or assuming tasks for trying different solutions. Furthermore, students must be able to engage in such work even when not under the direct supervision of the teacher. This is a specific skill, and is reflective of a more general classroom culture of productivity. Students are not born with such skills; they need to be explicitly taught, and practiced. When they are, they make a material contribution to the culture of productive engagement with high-level work and learning.

Cluster 2: Safe, Respectful, Supportive, and Challenging Learning Environment			
Indicators: ▪ Language of caring and respect between teacher and students and among students, and teacher's awareness of students' interests in and beyond school (2a) ▪ High levels of cognitive energy (2b) ▪ A safe environment for student risk taking (2a) ▪ High expectations for students' capabilities for learning (2b) ▪ Productive student engagement in small group work (2c) ▪ Students persevere, even in the face of challenges (2b)		*Evidence*: • Observation: o Interactions between teacher and students and among students o Student perseverance and commitment to learning o Student participation and productivity in partner, small group, and whole class work • Student surveys	
Unsatisfactory	**Basic**	**Proficient**	**Distinguished**
Interactions between teacher and students and among students are characterized by negativity, lack of support, low expectations, and low levels of student perseverance, through some combination of the following: • *The teacher uses disrespectful talk toward students; student body language indicates feelings of hurt or insecurity.* • *The teacher does not address disrespectful interactions among students, or the teacher's attempts to respond to disrespectful behavior are not successful.* • *The teacher displays no familiarity with, or caring about, individual students' interests or personalities.* • *The teacher conveys, to at least some students, that the mathematics work is too challenging for them.* • *Students exhibit little or no pride in their work; they abandon their efforts in the face of difficulty.* • *Students participate in only routine responses and tasks.* • *Students receive no support from their classmates.* • *Group work is unproductive; students either work independently or copy answers, or one student dominates the group's work.*	Interactions between teacher and students and among students are a mix of high and low support, moderate expectations, and modest levels of student perseverance, through some combination of the following: • *The quality of interactions between teacher and students, or among students, is uneven, with occasional disrespect.* • *The teacher attempts to respond to disrespectful behavior among students, with uneven results.* • *The teacher attempts to make connections with individual students, but student reactions indicate that the efforts are only partially successful.* • *The teacher conveys only modest expectations for students' mathematical learning abilities.* • *The teacher encourages students to persevere with challenging work; but only some do so, or they do so in a desultory manner.* • *Few students offer their ideas on questions that seem to entail intellectual risk.* • *Students offer assistance to classmates in a supportive manner when prompted by the teacher.* • *Group work is uneven: sometimes collaborative, sometimes not. The teacher makes intermittent attempts to support group processes.*	The classroom is characterized by interactions that are both supportive and challenging, with student perseverance in challenging work, through some combination of the following: • *Talk between teacher and students and among students is uniformly respectful, with little to no intervention needed by the teacher to correct disrespectful talk among students.* • *The teacher makes connections with individual students.* • *The teacher demonstrates a high regard for student abilities in learning mathematics.* • *Student work and conduct during a lesson indicate commitment to high quality; students persevere in understanding challenging content.* • *Students participate willingly and appear confident in offering their ideas in front of classmates.* • *Students spontaneously offer assistance to classmates in a supportive manner.* • *Students are productively engaged collaboratively with a partner or during small-group work.*	Classroom interactions indicate high levels of caring and respect, and student assumption of responsibility for the culture of civility, mutual support for work of high quality, and perseverance in achieving that quality, through some combination of the following, in addition to elements listed under "Proficient": • *Talk between teacher and students and among students is uniformly respectful, with no intervention needed by the teacher to correct disrespectful talk among students.* • *The teacher demonstrates knowledge and caring about the lives of students beyond school.* • *Student questions and comments indicate a desire for deep understanding of mathematics* • *Students take initiative in improving the quality of their work.* • *Students volunteer ideas, even when these ideas might seem to be unpopular among classmates.* • *Students recognize and express appreciation for the efforts of their classmates.* • *Group work is productive; groups take shared ownership of, and pride in, the products of their work. All members contribute to the group's work.*

Cluster 3: Classroom Management

A fundamental requirement for any productive classroom is that it runs smoothly. Teachers must establish efficient procedures for the completion of routine tasks, such as taking attendance, guiding transitions into work groups, distributing and collecting materials, and handling end-of-class dismissal. These procedures accomplish several essential purposes, are taken care of with a minimal loss of instructional time, and provide, for students, the security of familiar routines for students. Efficient routines convey to students that the teacher is in charge, though not a dictator, thus assuring them that they need not fear chaos.

Classrooms are, after all, crowded places; there are typically over 25 students, plus a teacher, in a relatively small space. This fact is a source of anxiety for many new teachers; they fear the large numbers of students in the classroom will overwhelm them, particularly if the students are physically larger than the teacher. What is to prevent, after all, an outright mutiny, with students simply refusing to comply with the teacher's directions? How to avoid chaos, with students doing whatever they choose, perhaps causing harm to themselves or other students? How can a teacher ensure that students actually *learn* anything? What is to guarantee that students will actually follow the rules, rather than just take charge? These are not unreasonable questions, and a new teacher's anxieties are understandable. In creating and then promulgating classroom routines and procedures, including behavioral norms, a teacher should keep in mind the principles that follow.

Routines and norms should be created with student participation.

Students, like other people, need to feel in control of their lives; they are quickly alienated by a teacher whose approach to classroom management is one of "This is how it is because I say so." Moreover, classroom routines are established not only to maintain an orderly environment, but also to solve real or potential practical problems. Thus, students will readily recognize that since they like to have a chance to speak in a discussion, the challenge is to work out an approach allowing everyone the opportunity to be heard. The same thinking applies to virtually all routines: the question "What would happen if we all just went for the door at the same time?" will elicit, even from young children, the recognition that the result would be chaotic—chairs could be overturned or some students knocked over. Next can come the question, "What might be some reasonable procedures for leaving the room?"

The attitude of the teacher in establishing routines and procedures is all-important. It's essential that the teacher convey to the students a concern to establish, with them, an environment in which important and interesting work can be accomplished. Therefore, routines and norms are needed for many activities: distributing and collecting materials, keeping a neat classroom, moving between large- and small-group activities, and so on. That is, the purpose of the routines is to maximize student learning; it's not because the teacher insists on control. This attitude permits the teacher to sincerely elicit student contributions.

Routines must be taught.

But even after students and the teacher have developed the routines and norms for how the class will operate, those routines must be taught and practiced. That is, teachers cannot simply assume that their students will automatically know what is intended by a direction such as "Move into your small work groups." Unless students have practiced a routine by which to accomplish such a task, the alternative, given the crowded nature of many classrooms, can be chaos. Thus, experienced teachers devote some time at the beginning of a year to actually *teach* the routines for all sorts of everyday classroom procedures: distributing and collecting materials, pushing chairs in at the end of class, and so on. Teaching routines is the same as teaching other skills: the routine is described, and students have a structured opportunity to practice it (for example, a transition to small groups) and do it again, incorporating feedback about the success of the first attempt. The same also applies to norms of behavior; they can be isolated, and role-played, so that students know what to expect when involved in a situation calling for a teacher to take corrective action. In this way, students are not caught off guard, or unprepared, by events.

It's likely that an observer can only infer from teacher directions and student actions whether routines were, in fact, established earlier in the year. Moreover, those teachers who are fortunate enough to have the assistance of volunteers or paraprofessionals in their classrooms have the additional challenge of ensuring that those individuals are productively engaged in making a substantive contribution to the life of the class.

Cluster 3: Classroom Management			
Indicators: ▪ Efficient procedures for non-instructional activities: taking roll, distributing and collecting materials, making transitions, etc. (2c) ▪ Clear guidelines for student work when it is unsupervised, e.g., in small groups (2c) ▪ Evidence of clear standards of conduct, understood by the students, monitored by the teacher, corrected successfully (when necessary) by teacher or students, or both (2d) ▪ Physical environment supportive of learning activities (2e) ▪ Productive contribution to the class by volunteers and paraprofessionals (2c)		*Evidence*: • Observation: o Routines o Student conduct o Physical environment	
Unsatisfactory	**Basic**	**Proficient**	**Distinguished**
The classroom environment is disorganized and chaotic, through some combination of the following: • *Classroom procedures for transitions and other non-instructional duties are either absent or inefficient, resulting in the loss of much instructional time.* • *Groups not working with the teacher are not involved in productive work.* • *No standards of conduct appear to have been established, or the teacher does not monitor student behavior, or, when noticing student misbehavior, appears helpless to do anything about it.* • *There are physical hazards in the classroom, endangering student safety.* • *Volunteers and paraprofessionals have no defined role and may be idle much of the time.*	The classroom environment is a little rough, through some combination of the following: • *Procedures for transitions, materials, and other non-instructional duties seem to have been established, but their operation is rough or inconsistent, resulting in some loss of instructional time.* • *Small groups are only intermittently engaged while not working directly with the teacher.* • *Standards of conduct appear to have been set, but the teacher's attempts to maintain order meet with uneven success, or the teacher's response to student misbehavior is inconsistent: sometimes very harsh, other times lenient.* • *The physical environment is not an impediment to learning but does not enhance it.* • *Volunteers and paraprofessionals participate but require frequent supervision or their work is not well integrated with classroom activities.*	The classroom functions smoothly and efficiently, through some combination of the following: • *Efficient procedures have been established for non-instructional activities, such as distribution and collection of materials and supplies, transitions to other grouping patterns, etc. resulting in minimal to no loss of instructional time. Students carry out procedures with little or no teacher direction.* • *All students are productively engaged during small-group work, a fact indicating established procedures.* • *The teacher regularly monitors student behavior; student behavior is generally appropriate. When needed, the teacher's response to misbehavior is effective.* • *The classroom is arranged to support the instructional goals and learning activities.* • *Volunteers and paraprofessionals work with minimal supervision in synch with classroom goals.*	The classroom functions seamlessly, through some combination of the following, in addition to elements listed under "Proficient": • *Students take the initiative with their classmates to ensure non-instructional routines run smoothly.* • *Students ensure productive small-group work by, for example, assigning roles.* • *The teacher's monitoring of student behavior is seamless and preventive, accomplished through nonverbal means; student behavior is entirely appropriate.* • *Productive classroom norms are well established, and students as well as the teacher act to maintain them.* • *Students take the initiative to contribute to and adjust the physical environment so it supports learning for all.* • *Volunteers and paraprofessionals take initiative in their work in the class, a fact indicating clear roles and training.*

Cluster 4: Student Intellectual Engagement

Student engagement is at the very heart of good teaching; it is typically the first item educators identify when invited to describe the classroom of a teacher whom they consider an expert. However, the term *engagement* does not have a single, or a simple, definition. First, intellectual engagement is not the same as being busy or on task; it's quite possible for students to be occupied doing work—for example, completing a poorly designed worksheet—that does not represent new learning. Furthermore, physical activity is not sufficient; an activity might involve students in working with physical materials - such as Cuisenaire Rods - but doing so in a formulaic manner. The key to student engagement is not physical, but mental, activity. A task might be "hands-on." But in order to qualify as intellectual engagement, it must be "minds-on." School, in other words, from the point of view of students, is not a spectator sport. Therefore, it's essential to maximize the extent to which students are involved in intellectual activity, such as exploring new mathematical ideas, seeing relationships, making connections, or formulating and testing hypotheses.

A useful rule of thumb that indicates the degree of student intellectual engagement is the answer to the question, "Who's doing the work?" When students listen while the teacher makes a presentation, demonstrates a procedure, or applies a rule, their role may be entirely passive; they may be simply watching while the teacher performs. However, a teacher may present new mathematical ideas in such a way that students are invited to connect new information with prior understanding or predict outcomes of a problem at hand. When teachers structure lessons in such a way that students are intellectually active, those students deepen their understanding of concepts and generate new knowledge. This process involves thinking. Thus, a variation on the maxim "Who's doing the work?" is "Who's doing the thinking?" Only when students are actively thinking (as part of an exploration of mathematical ideas, engaging in a discussion led by the teacher or with classmates or completing a task) can they be said to be intellectually engaged.

In addition to students being engaged in thinking; they can also become aware of their own cognitive processes: that is, teachers can engage students not only in cognitive work, but also in *metacognitive* work. How did they arrive at a certain answer? What's the evidence for it? In making an error in solving a problem what was the trajectory of their thinking? Where did it go off track? Can they retrace their steps and find the error? These latter questions deal with the process of thinking and are highly transferable to other situations, and indeed to other subjects. They enable students, when they encounter difficulty through, for example, arriving at a false conclusion, to retrace their steps and take corrective action.

It should also be noted that student engagement in learning does not always appear tidy; when students are wrestling with a new concept or making connections between new mathematical ideas and previously learned material, they may make a few false starts or pursue what turns out to be a dead end before making a course correction. It's challenging for some teachers to allow their students to engage in this *productive struggle*, but the resulting understanding is satisfying to students, empowering them as learners. Furthermore, the learning that results is deeper, more flexible, and longer lasting and solidifying their comprehension.

A mathematics lesson in which students are engaged usually has a discernible structure: a beginning, a middle, and an end, with scaffolding provided by the teacher or by the activities themselves. The teacher organizes student tasks to provide cognitive challenge and encourages students to reflect on what they have done and what they have learned. That is, the lesson has closure, in which the teacher encourages students to derive important learning from the learning tasks, from the discussion, or from what they have read.

Visitors have no difficulty recognizing a classroom with high levels of student cognitive engagement. There is palpable (almost electric) energy in the room, as students display commitment to their work, and are eager to explain their accomplishments to visitors. This is not the busy work of students complying with a teacher's requests for them to complete assignments; in a classroom in which students are pursuing their own goals, their work is self-directed, and the environment is vibrant.

For teachers, there are two critical aspects to teaching for student intellectual engagement: designing (or locating) and managing rich learning tasks within the sequence of an arc of learning, and skillfully using student discourse.

Rich learning tasks

Designing (or identifying) suitably demanding learning tasks for students is one of the most challenging aspects of mathematics teaching, since a task that is challenging for one student may be routine for another. One can analyze the cognitive demand of a task; whether the task is suitably rigorous, or appropriate, for an individual student is determined by the level of knowledge and cognitive development of the student. Thus, a task, in and of itself, is not rigorous or routine; what makes it rigorous or routine is the gap between the

demands of the task and the current capabilities of the students who are asked to complete it. If the gap is small or nonexistent, the task is routine and boring; if the gap is too great, the task may be overwhelming. Like Goldilocks's porridge, the gap should be "just right." One technique to address this challenge is to assign tasks with a low floor and a high ceiling—that is, tasks that are accessible to all students, but that, through their expansion, or through the teacher's asking a more demanding follow-up question, can challenge the more-advanced students in the class. Employing this technique is not a simple matter and is developed only after considerable teaching experience.

Another characteristic of rich learning tasks relates to being "group-worthy," that is, they invite multiple perspectives that may be represented by the different students working together in groups. Much classroom activity, after all, takes place in pairs or small groups, with the teacher playing a mediating, rather than a direct "teaching," role. Tasks that are suitable for collaborative work are typically those with more than one correct answer, or more than one method to arrive at the single correct answer. Such tasks enable students with different strengths, or who might see a situation in a different way, to make a contribution to the overall effort. In order for such work to be productive, of course, students must have acquired the skills of collaboration described in Cluster 2.

Student discourse

Questioning and discussion is used to deepen student understanding (rather than serve as recitation, or a verbal "quiz"). Effective teachers use divergent as well as convergent questions, framed in such a way that they invite students to formulate hypotheses, make connections, or challenge previously held views. These teachers are especially adept at responding to and building on student responses and making use of their ideas.

Class discussions should be animated, engaging students in important mathematical ideas and promoting the use of precise language to deepen and extend understanding. These discussions may be based around questions formulated by the students themselves. Furthermore, when a teacher is building on student responses to questions (whether posed by the teacher or by other students), students are challenged to explain their thinking, to critique the reasoning of others, and to cite specific evidence to back up a position. This focus on argumentation forms the foundation of logical reasoning, a critical skill in all disciplines.

Cluster 4: Student Intellectual Engagement	
Indicators: • The content is seen as worthwhile, important, and interesting (2b) • Content is presented in a manner that engages students in thinking and reasoning (3a) • Learning tasks require students to engage intellectually, to *think;* some may involve productive struggle (3c) • Questions/discussions involve higher-order cognitive activity; students have time to develop their ideas and productive mathematical practices/habits of mind (3b).) • The lesson has a recognizable structure, with time for reflection and closure (3c) • Students explain their thinking and question the thinking of others (3b)	*Evidence*: • Classroom observation: o The quality of teacher explanation of content o The structure of the lesson o The nature of the work students are doing o The nature of student discourse and class discussion o Student metacognition • Student worksheets or activities • Samples of student work

Unsatisfactory	**Basic**	**Proficient**	**Distinguished**
The level of intellectual engagement on the part of students is low, through some combination of the following: • *The teacher conveys no energy for the importance of the learning goals and assignments.* • *Mathematical concepts are presented in a didactic manner, with no invitation for students to think.* • *Learning tasks require only recall or are presented as a single procedure to be followed.* • *The teacher's questions are rapid-fire and convergent, with a single correct answer, and do not invite student thinking.* • *All discussion is between the teacher and individual students; students are not invited to speak directly to one another.* • *The teacher does not ask students to explain their thinking; they are asked only to produce answers.* • *Few students are involved in the activities and discussions.* • *The lesson has no recognizable structure; it's a random series of events.*	The level of intellectual engagement on the part of students is modest, through some combination of the following: • *The teacher displays little energy for the lesson's purpose or assignments.* • *The teacher's explanation of the mathematics content consists of a monologue, with minimal participation or intellectual engagement by students.* • *Learning tasks are so highly scaffolded that the result is a single pathway to completion.* • *Student suggestions for multiple pathways to solving problems are quickly dismissed* • *Some of the teacher's questions invite student thinking.* • *The teacher attempts to provide time for students to formulate their ideas; some make productive use of this time.* • *The teacher invites students to respond directly to one another's ideas, but few students do so.* • *The teacher asks students to explain their mathematical reasoning and cite specific reasons, but only some students attempt to do so.* • *About half the students are involved in activities and discussions.* • *The lesson has a recognizable structure, although parts of it may be rushed, while others drag.*	The classroom is a cognitively busy place, with students encouraged to use their minds, through some combination of the following: • *The teacher exhibits energy for the topic and conveys its importance.* • *The teacher's explanations of mathematical concepts invite student intellectual engagement.* • *Learning tasks, some of which are "group - worthy," demand higher-order thinking, inviting students to take initiative, and may involve productive struggle.* • *Multiple pathways to solving problems are encouraged, with students inventing different approaches. The teacher guides the class towards those methods that are most efficient and most relevant for future work.* • *Many of the teacher's questions are open-ended, or have multiple correct answers, inviting students to think. (When low-level questions are used, they are part of targeted fluency building or else provide scaffolding for new learning.)* • *Wait time is used productively; students engage in thoughtful reflection during discussion.* • *Students direct their comments to one another during full class discussions; there is lively discussion during small-group work.* • *Students are asked to explain their thinking, citing specific arguments and mathematical reasons; most do so.*	The classroom is a cognitively vibrant place, with students encouraged to use their minds, through some combination of the following, in addition to elements listed under "Proficient": *The teacher explains content clearly, ensuring the mathematics is explicit, and inviting student predictions to bring content to life.* • *Students appear to relish challenging tasks and push their classmates with extended questions.* • *Students initiate higher-order mathematical questions.* • *Students extend the discussion, enriching it.* • *Students invite comments from their classmates during a discussion and push their classmates with extended questions in both small group and whole class contexts.* • *Students pursue mathematical questions that they find interesting, or use alternative strategies and tools they find meaningful.* • *Students themselves ensure that all their classmates are involved in the activities and discussions.* • *Students have an opportunity for reflection and closure on the lesson to consolidate their understanding.* • *Students build on each other's ideas and make conjectures/connections aimed at either deeper conceptual understanding or at connecting procedures to underlying*

Cluster 4: Student Intellectual Engagement			
		• *Virtually all students are involved in the activities and discussions.* • *The lesson has a clear structure, with time for students to engage in thoughtful participation in discussions and learning tasks.*	*concepts.* • *Students make independent use of explanations, arguments, and mathematical representations.* • *Students help classmates connect their strategies to one another*

Cluster 5: Successful Learning by All Students

It is not sufficient for teachers to engage in an activity called teaching; they must ensure that students learn. That is, one way of defining *teaching* is as "that which causes student learning." While this appears an obvious statement, educators frequently overlook it as they attempt to codify *good teaching* in ways that focus exclusively on the actions of teachers without considering the success of those efforts in ensuring student learning.

Teachers recognize that all mathematics learning is complex, involving the interplay of conceptual and procedural knowledge, facts and processes, dispositions and habits of mind. Students don't "master" all of these in the same way, or in the same sequence, and they enter any lesson with their own strengths and areas for growth. However, every lesson and longer unit has a focus, and it's in that area of focus that teachers must be able to articulate, and make specific plans to address, what they intend students to learn.

Ascertaining whether students have, in fact, learned what was intended requires the design (or adoption) of summative assessments aligned to those outcomes (so that the teacher can take corrective action before moving on), and formative assessments to be used, on short notice, during the course of a unit or lesson. This requires sophisticated record-keeping systems. In addition, in order for teachers to modify their approach to ensure that all students are making progress towards the instructional purposes of the lesson, they must not only be aware of resources (in the school or, more broadly, in the district or the community) that can be brought to bear. They; they also must be committed to do what is needed to help every student succeed.

Traditionally, teachers did not ascertain the extent to which their students had learned the material being taught until they had completed an instructional unit; indeed, the assessment (usually a test of some type) signaled the end of instruction, students' work was graded, and the class went on to the next unit. In this approach, teachers could know whether or to what extent their students *had* learned but could not ensure that they did so. Fortunately, many teachers now employ a subtler approach, one designed to shape instruction during the course of a lesson or unit. Teachers monitor students' responses and activities constantly, monitoring the "pulse" of the class frequently during a lesson and making revisions to their approach when needed. These changes might take the form of making a slight modification in the pace of an activity or in the activity itself, based on students' indications of lack of comprehension (too challenging) or boredom (too easy.)). Such monitoring occurs constantly and is not specifically planned.

However, other techniques used by teachers may be, indeed must be, planned in advance. An important way to gather information about what students are learning is to ask students to communicate their developing ideas and thinking, and to listen carefully to their responses. Alternatively, students' responses to a carefully-crafted question, with their answers written on whiteboards and held up for the teacher to see, provide important diagnostic information about individual students' level of understanding This approach is most powerful when students' incorrect answers reveal information about their thinking, or the types of misconceptions they hold. To the extent that such formative assessment is integrated into instruction, teachers can make minor modifications as needed. Sometimes students provide such indications explicitly; they ask clarifying questions, for example. On other occasions, however, the indications are much more subtle or camouflaged, for example a quizzical look. Whether implicit or explicit, teachers must be alert to such indications of student understanding.

Another important mechanism to ensure students' success is arranging for them to receive specific and timely feedback on their efforts. The teacher can provide this feedback, of course. But it can also be supplied by other students (as when they challenge – —respectfully – —the thinking of their classmates), or by the instructional activities themselves. For example, the solution to a problem in mathematics may simply "not "work." Whatever the source of the feedback, students come to realize that learning is a process of continual iteration; it's never complete.

Families, too, can be allies in a teacher's quest to ensure student success. They have, after all, known the students for a longer time than has the teacher, and can provide insight into the students' lives and interests beyond school. Such information can be invaluable to a teacher in planning instruction and responding to individuals. Skilled teachers keep parents and guardian abreast of students' success in school, and draw on the insights of their families in how to enhance that success.

Attention to every student's learning is grounded in some important assumptions, namely, that the students are capable of high-level learning and that the teacher has the necessary skill, resources, and attitude to enable them to succeed. These beliefs are fundamental. If teachers lack a strong sense of efficacy, then they will be inclined to give up easily when students experience difficulty (as virtually all students do at some points). In such cases, teachers find other factors on which to place the "blame" for students' struggles: their backgrounds ("His parents are getting a divorce"), the perceived weaknesses of older siblings ("Her brother never could do fractions either"), the lack of skill of

a previous teacher ("They should have learned this last year"), or the inadequacy of the adopted materials ("This textbook is terrible"). Therefore, teachers' ensuring the learning of every student is a reflection of their confidence that they can teach well and that their students are capable of high-level learning.

Cluster 5: Successful Learning by All Students

Indicators:	*Evidence*:
• Both summative and formative assessments, aligned to learning outcomes, have been planned (1f) • The teacher monitors student learning during the lesson (individuals and groups) through a variety of means (3d) • Students receive specific feedback on their work from the teacher, the activities themselves, or other students (3d) • If necessary, the teacher modifies the lesson to ensure that students "get it," drawing on other resources as needed (1d, 3e) • The teacher's records permit detailed analysis of learning by individuals and groups of students (4b) • The teacher enlists, as appropriate, the engagement of families in student learning (4c) • In reflection, the teacher assumes responsibility for student learning (4a)	• Planning documents: formative and summative assessments • Observation: monitoring, feedback, adjustment • Artifacts documenting record keeping, communication with families • Reflection: comments on individual students' learning

Unsatisfactory	Basic	Proficient	Distinguished
The teacher makes no attempt to ensure the learning of all students, through some combination of the following: • *Summative assessments are poorly aligned with the learning outcomes.* • *No formative assessments have been designed for use during the lesson.* • *The teacher makes no effort to determine whether students understand the content of the lesson or ignores indications of student boredom or lack of understanding.* • *Feedback to students is only global, such as, "Good job, everyone."* • *The teacher makes no attempt to adjust the lesson, even when such action is clearly needed.* • *The teacher conveys to students that when they have difficulty learning it is their fault.* • *Record-keeping systems are in disarray.* • *Families are unaware of their children's progress.* • *In reflecting on the lesson, the teacher cites the extent to which students were busy or were well behaved, with no comments about the extent to which they achieved the intended outcomes.*	The teacher makes sporadic or inconsistent attempts to ensure the learning of all students, through some combination of the following: • *Only some of the instructional outcomes are addressed in summative assessments.* • *Plans refer to the use of formative assessments but with no specificity.* • *The teacher requests global indications of student understanding, such as, "Any questions?"* • *Feedback to students is neither specific nor oriented toward future improvement of work.* • *The teacher's efforts to modify the lesson are only partially successful.* • *The teacher conveys to students a sense of his or her own responsibility for their learning but also uncertainty about how to assist them.* • *The teacher may mention typical student errors, but not deal with them when they arise, in a diagnostic way.* • *The teacher maintains school-required record-keeping systems but does little else to inform families about student progress.* • *The teacher communicates sporadically with families regarding student learning.* • *In reflecting on the lesson, the teacher cites only limited evidence of student attainment of the instructional goals with an emphasis on other*	The teacher makes genuine attempts to ensure the learning of all students, through some combination of the following: • *All the learning outcomes have a method for summative assessment, differentiated, as needed, for students with different learning goals.* • *Plans include specific formative assessments for use during instruction.* • *The teacher monitors student learning through a variety of means, including using specifically formulated questions, differentiated as needed, to elicit evidence of student understanding.* • *Feedback includes specific and timely guidance on how students can improve their learning.* • *The teacher makes productive changes to the lesson plan in response to evidence of student difficulties.* • *The teacher conveys to students that s/he has other approaches to try when the students experience difficulty.* • *The teacher maintains a coherent record-keeping system on student learning and regularly sends home information about student progress.* • *The teacher communicates regularly with families regarding student learning.*	The teacher indicates a deep commitment to the learning of all students, through some combination of the following, in addition to elements listed under "Proficient": • *The teacher constantly "takes the pulse" of the class; monitoring of student understanding is sophisticated and continuous and makes use of strategies to elicit information about individual student learning.* • *Students monitor their own learning, either on their own initiative or as a result of tasks set by the teacher.* • *High-quality feedback comes from many sources, including other students; it is specific and focused on improvement.* • *The teacher actively encourages two-way communication with families regarding student learning.* • *In reflecting on the lesson, the teacher has specific ideas about how the lesson could be improved. The teacher cites student assessment data that will be taken into account in future planning.*

Cluster 5: Successful Learning by All Students			
	factors, such as whether students were busy or were well behaved.	• *In reflecting on the lesson, the teacher cites specific examples of student attainment of the instructional goals.*	

Cluster 6: Professionalism

Schools are, first of all, environments to promote the learning of students. But they are also places for the intellectual engagement of teachers, so that they can better promote the learning of their students. Schools are, in other words, learning organizations for teachers, and full potential is realized only when teachers regard themselves as members of a professional community. This community is characterized by mutual support and respect, as well as by recognition of the responsibility of all teachers to be constantly seeking ways to improve their practice and to contribute to the life of the school and to the broader professional community. Inevitably, teachers' duties extend beyond the doors of their classrooms and include activities related to the entire school or larger district, or both. These activities include such things as service on school and district mathematics curriculum committees or engagement with the parent-teacher organization. With experience, teachers assume leadership roles in these activities or others, and in their school communities in general.

As in other professions, the complexity of teaching requires continued growth and development in order for teachers to keep their knowledge and skills current. Continuing to stay informed and increasing their skills allows teachers to become ever more effective, and to exercise leadership among their colleagues. Mathematics has not, as a discipline, changed, but our professional understanding, as to how best to sequence the different mathematical topics, and how best to help students understand them, is in constant evolution. Thus, educators must constantly refine their understanding of how to engage students in learning; thus, growth in content and content-specific pedagogy is essential to good teaching. And to the extent that information technology is an aid to student mathematics learning, it's essential for teachers to stay abreast of developments in that area as well.

Networking with colleagues through such activities as joint planning that includes pre-solving problems and anticipating student responses, study groups, and lesson study provides opportunities for teachers to learn from one another. In particular, sharing perspectives while jointly examining student work can provide invaluable insight, that is not available in any other way, into the cognitive processes of individual students who may have wrestled with mathematical ideas. Furthermore, teachers' willingness to invite colleagues into their classrooms to observe their teaching serves to make practice public and to generate valuable conversations about "problems of practices." These activities allow for job-embedded professional development. In addition, professional educators increase their effectiveness in the classroom by belonging to professional organizations (at the regional, state, or even national level), reading professional journals, attending educational conferences, and completing workshops or university classes. As they gain experience and expertise, educators find ways to contribute to their colleagues and to the profession.

Expert teachers also demonstrate professionalism in service both to students and to the profession. Teaching at the highest levels of performance requires that teachers remain focused on students, putting them first regardless of how this stance might challenge long-held assumptions, past practice, or simply an easier or more convenient procedure. For example, dialogue around the issues surrounding the appropriate use of mathematics homework is certain to be spirited, and reveal teachers' deep belief about student learning and how best to support it.

Accomplished teachers have a strong moral compass and are guided by what is in the best interest of each student, even when this ethos involves challenging long-established school policies or procedures. They display professionalism in a number of ways. For example, they conduct interactions with colleagues in a manner notable for honesty and integrity. Furthermore, they know their students' needs and can readily access resources with which to step in and provide help that may extend beyond the classroom. Seeking greater flexibility in the ways school rules and policies are applied, expert teachers advocate for their students in ways that might challenge traditional views and the educational establishment. They also display professionalism in the ways they approach problem solving and decision making, with student needs constantly in mind. Finally, accomplished teachers consistently adhere to school and district policies and procedures but are willing to work to improve those that may be outdated or ineffective.

Cluster 6: Professionalism			
Indicators: • Collaboration with colleagues for joint planning, and school/district and community initiatives (4d) • Active engagement in workshops, courses, and activities to improve practice (1d, 4e) • Integrity and honesty in dealing with colleagues and parents on behalf of students (4f)		*Evidence*: • Artifacts documenting: o Contributions to school life and the professional culture o Professional learning o Other professional activities • Feedback and surveys from colleagues and supervisors	
Unsatisfactory	**Basic**	**Proficient**	**Distinguished**
The teacher makes no attempt to continue with professional learning or engage with the professional community to advance the interests of students, through some combination of the following: • *The teacher's relationships with colleagues are characterized by negativity or lack of trust.* • *The teacher avoids involvement both in school activities and in district and community projects.* • *The teacher ignores or avoids opportunities to participate in activities for professional learning.* • *The teacher declines to participate in team and departmental decision making, except when required by superiors.* • *The teacher does not prioritize the needs of students and operates in a self-serving manner.* • *The teacher ignores school and district regulations.*	The teacher makes sporadic or inconsistent attempts to continue with professional learning or engage with the professional community to advance the interests of students, through some combination of the following: • *The teacher has cordial relationships with colleagues and is trusted by them.* • *When asked, the teacher participates in school activities as well as district and community projects.* • *The teacher participates in professional activities when they are required or provided by the district.* • *The teacher participates minimally in team and departmental decision making.* • *The teacher notices the needs of students but is inconsistent in addressing them.* • *The teacher minimally complies with school and district regulations.*	The teacher makes genuine attempts to continue with professional learning and to engage with the professional community to advance the interests of students, through some combination of the following: • *The teacher has supportive, collaborative, and trusting relationships with colleagues and is known for having high standards of integrity.* • *The teacher frequently volunteers to participate in school events and in school, district, and community projects.* • *The teacher seeks opportunities for continued professional development.* • *The teacher actively participates in team and departmental decision making.* • *The teacher actively addresses student needs and actively works to provide opportunities for student success.* • *The teacher completely complies with the spirit, as well as the letter, of school and district regulations.*	The teacher indicates, through various actions and statements, a deep commitment to continuing professional learning and engagement with the professional community to advance the interests of students, through some combination of the following, in addition to elements listed under "Proficient": • *The teacher takes initiative and a leadership role in organizing collaborative projects.* • *The teacher regularly contributes to, and leads, significant district and community projects.* • *The teacher takes a leadership role in finding opportunities for continued professional development and in contributing to professional organizations.* • *The teacher takes a leadership role in team and departmental decision making, and enjoys the trust of colleagues in terms of honesty, integrity, and confidentiality.* • *The teacher makes a concerted effort to ensure opportunities are available for all students to be successful, even when these efforts challenge school or district policies.* • *The teacher makes material suggestions for the improvement of school and district regulations.*